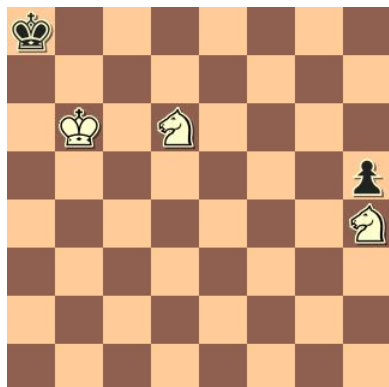
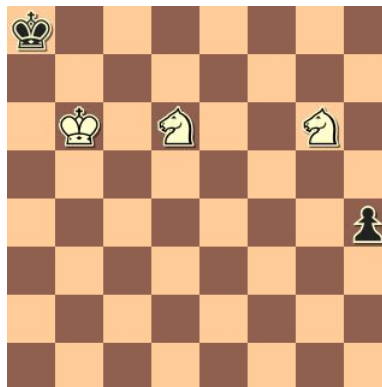


Alexei Troitzky (1866-1942) is a well known Russian composer of chess studies, and by most accounts the best composer who ever lived. In 1937 he published the first of a projected multi-volume collection of his studies, but World War II intervened - Germany invaded Russia, and the work was never finished. Troitzky died of starvation in August 1942, during the siege of Leningrad.

The single book that we have, Collection of Chess Studies, contains a special supplement devoted to endings with Knights against pawns. Those positions then lead to a section covering the last and most difficult "basic" checkmate in chess, a King and two Knights versus King and pawn. Here's the idea, illustrated by Capablanca as "Example 29" in Chess Fundamentals.

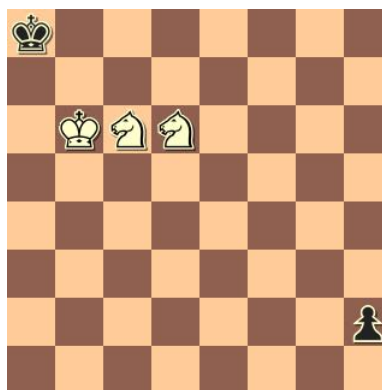


1.Ng6 h4

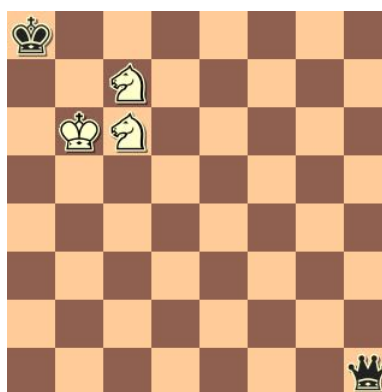


White cannot take the pawn because the game will be drawn, as explained before. - Capablanca

2.Ne5 h3 3.Nc6 h2

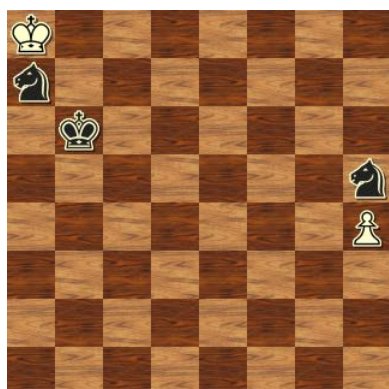


4.Nb5 h1=Q 5.Nc7#



Very pretty, and not a mere abstraction either, as this checkmate is seen more often than we imagine. Tennessee's own IM Ron Burnett has been here twice, and there's an Australian player, IM Stephen Solomon, who has faced it three times in FIDE games. The list of Grandmasters involved in this ending is impressive, and not all have been successful at checkmate.

I've had this ending twice myself; in fact, the conclusion of my first encounter with two Knights versus a pawn is a mirror of Capa's example, with colors reversed. From the game Barton-Pickard, Memphis 1982:



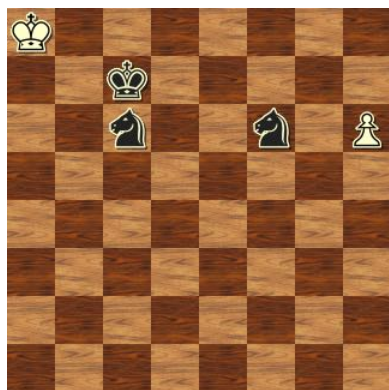
Barton-Pickard



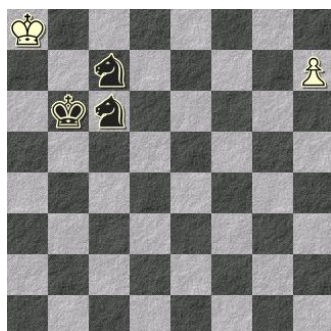
1...Nf6 2.h5 Nc6 Smothering the King, and the reason that White must have a pawn.



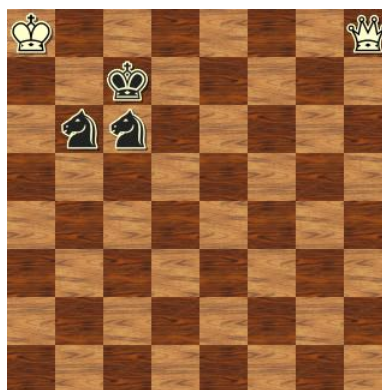
2.h6 Kc7



3...Nd5 4.h7 Nc7#

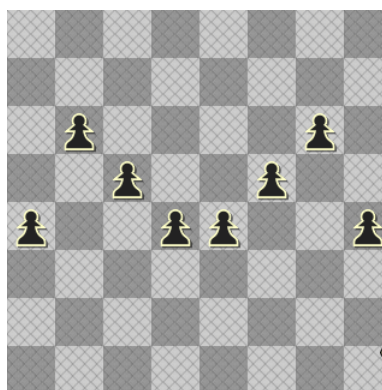


3.h7 Nd7 4.h8=Q Nb6#



The complete score was published in Tennessee Chess News, Oct.-Nov. 1982, page 48, for those who like digging into regional [archives]. The takeaway, however, is that these checkmates are being replicated almost every day - largely due to the work of Troitzky. And yet there's much left to be discovered about two Knights versus a pawn, even with those all-knowing 5-piece endgame tablebases that see every possible outcome!

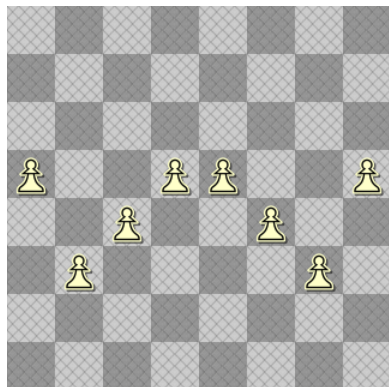
Partly to blame for any theoretical blank spots, however, is Troitzky himself. His supplement amounts to a bare 40 pages, hard to follow in the first place, and it's still in old English descriptive notation. He spends a lot of time showing proper corner positions, how to drive the weaker King along the edge, and much more besides. At last he gets to actual examples, studies really, but here we'll survey only one offshoot of our composer's analysis, the so-called Troitzky Line:



Quite a symmetrical diagram, but what does it mean? Well, the Troitzky Line is a tool used to examine our checkmate challenge of two Knights versus a pawn. The story is that if Black's pawn is blockaded on a square no further forward than the line a4-b6-c5-d4-e4-f5-g6-h4, then White can deliver mate.

An internet search for "Troitzky Line" will yield plenty of images like the one presented, all touting the same formula - blockade along the Troitzky Line and win. Sounds simple, right?

Keep in mind also that chess studies always show White as the stronger side, but in tournament play we have to imagine the reverse, in which the line a5-b3-c4-d5-e5-f4-g3-h5 wins for Black. Here's a Troitzky Line you won't see on the web.



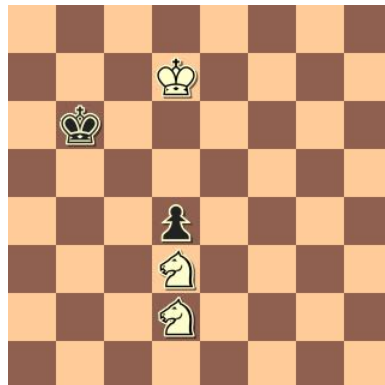
Notice that the board can be divided in half, so that whatever is true of the four Kingside pawns is also true of the Queenside pawns. There are also four quadrants of 16 squares each, zones useful in analyzing this endgame. Real work on two Knights versus a pawn is hardly possible without the Troitzky Line to map a way forward.

The funny thing is, our composer never mentioned a Troitzky Line nor did he illustrate any such line. It was derived much later, using remarks and individual studies to build the figure above. Nowadays the Troitzky Line has become almost a meme, passed from book to book and web page to web page, with little regard for details. Block a pawn on the line and checkmate!

But is that so? I've got some questions about this Troitzky Line, seeing its late arrival on the chess scene. First, is the line actually valid? That is, can White deliver checkmate from the Troitzky Line in every case, under any circumstance in any corner? The answer is negative, as plainly analyzed by Troitzky himself.

White can indeed checkmate, but there are plenty of caveats and loopholes along the way - things they never taught us in grade school.

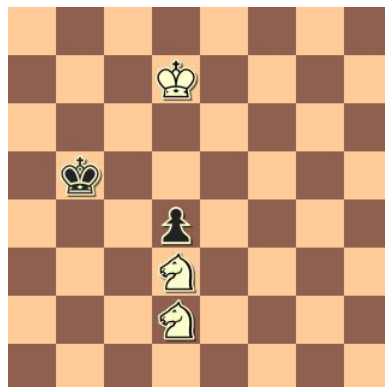
Among those countless Troitzky Line memes there's hardly a hint of exceptions or exclusions, no conditions, no "ifs, ands, or buts" - only happy promises of victory. The actual situation, however, is quite different.



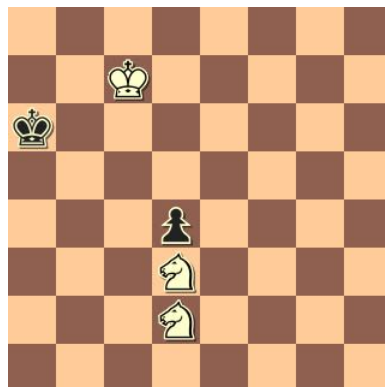
Black's d-pawn is blockaded on the Troitzky Line, and White wins with either side to move. One little fact, however, is never mentioned - facing best defense mate can only be achieved in the h1 corner! Yes, Black can fall into checkmate elsewhere but it can only be enforced on the h1 square.

The upper left quadrant is the most difficult for White to handle, as he must begin far from the mating zone. Troitzky shows that other starting positions always lead here, and will continue inevitably as follows.

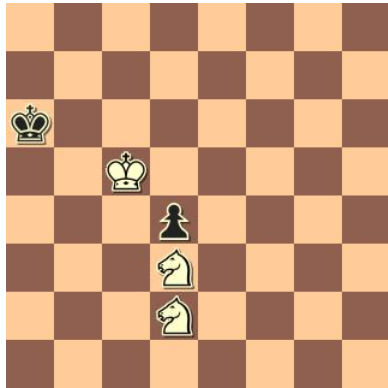
1...Kb5



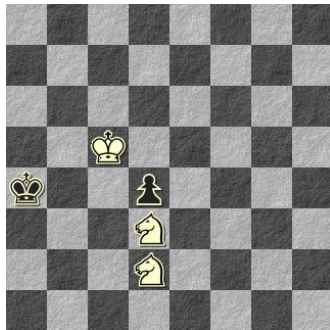
2.Kc7 Ka6



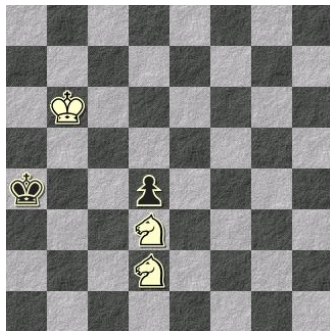
3.Kc6 Ka5 4.Kc5 Ka6



If 4...Ka4



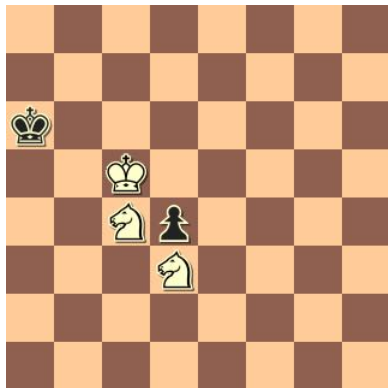
then 5.Kb6 and it's mate in 11 moves.



Mate in 11.

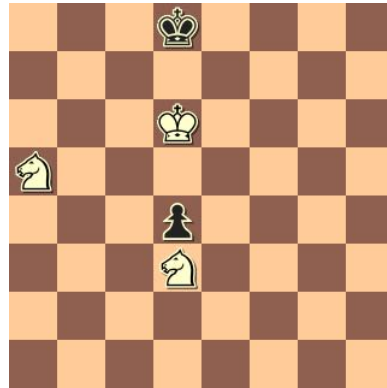


5.Nc4

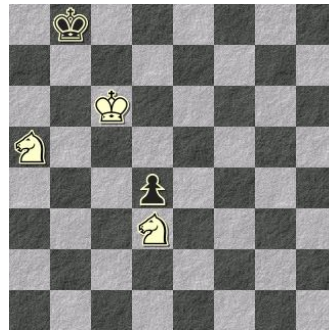


Mate in 27 moves, says Troitzky, though later analysis by N. Kopayev shortened the solution by four moves. Black must be driven around the edges of the board towards h1.

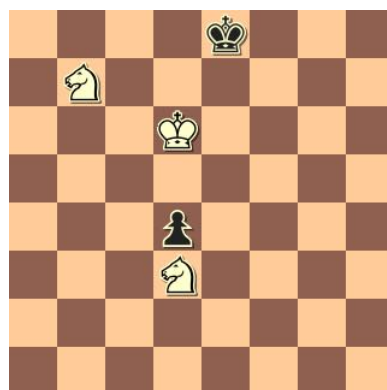
5...Kb7 6.Kd6 Kc8 7.Na5 Kd8



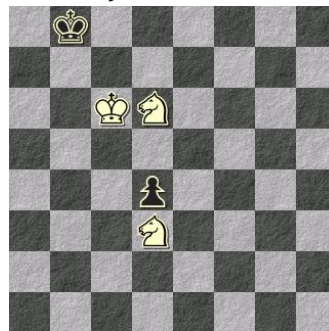
7...Kb8 8.Kc6



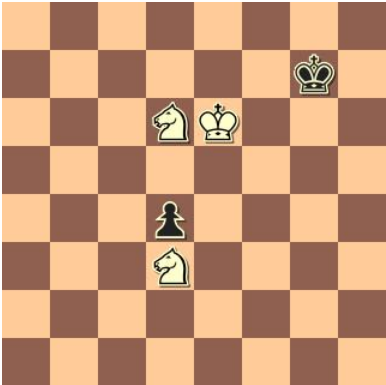
7.Nb7+ Ke8



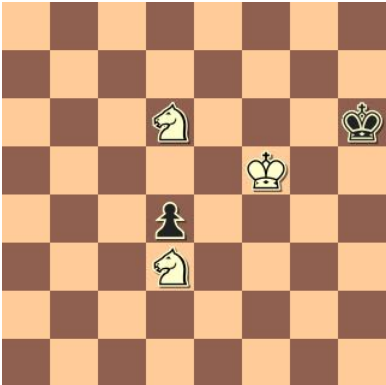
8...Kc8 9.Kc6 Kb8 10.Nd6



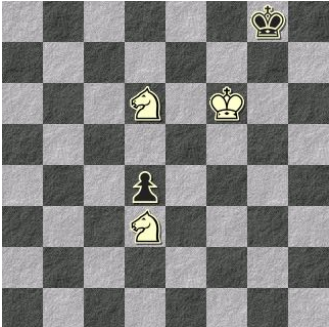
8.Ke6 Kf8 9.Nd6 Kg7



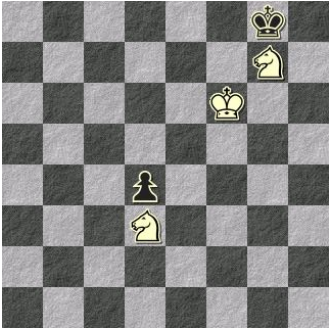
10.Kf5 Kh6



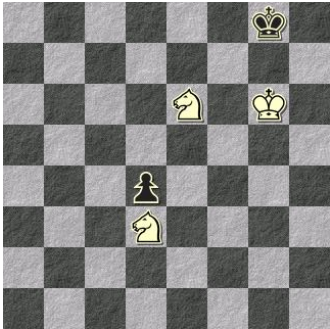
11...Kf8 12.Kf6 Kg8



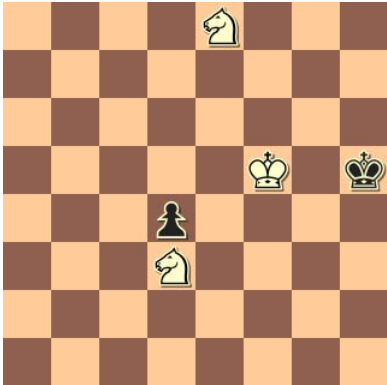
13.Nf5 Kf8 14.Ng7 Kg8



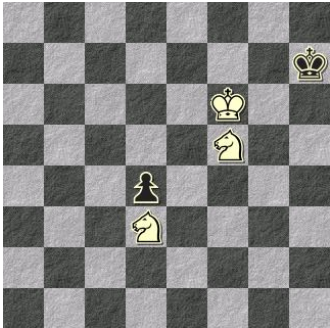
15.Ne6 Kh7 16.Kg5 Kg8 17.Kg6



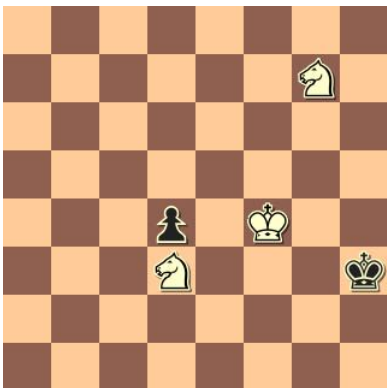
11.Ne8 Kh5



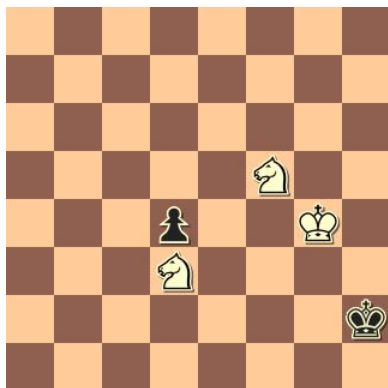
12...Kh7 13.Kf6 Kh6 14.Ng7 Kh7 15.Nf5



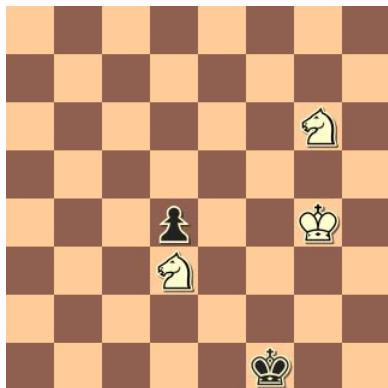
12.Ng7+ Kh4 13.Kf4 Kh3



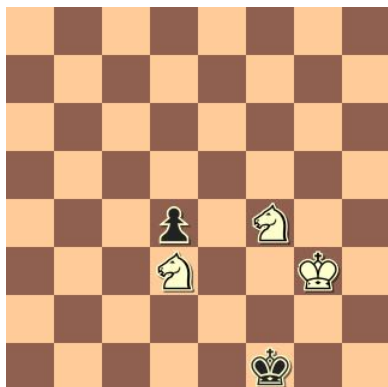
14.Nf5 Kg2 15.Kg4 Kh2



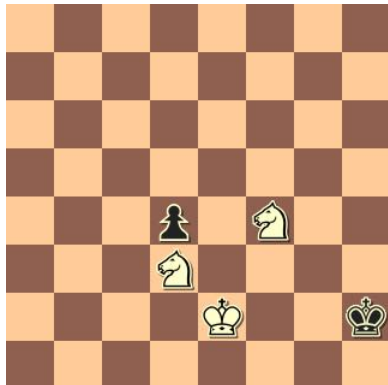
16.Nh4 Kg1 17.Ng6 Kf1



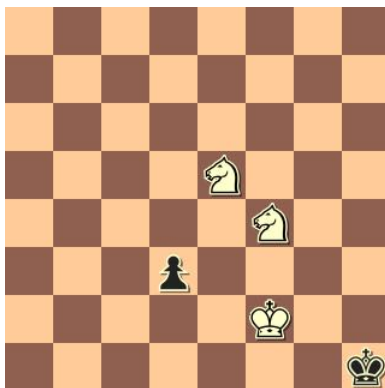
18.Ngf4 Kg1 19.Kg3 Kf1



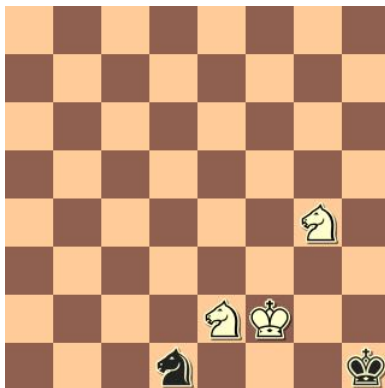
20.Kf3 Kg1 21.Ke2 Kh2



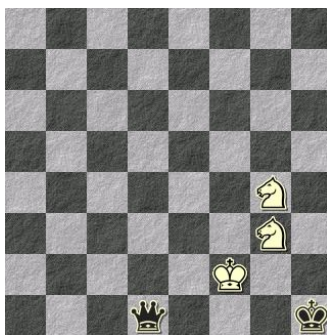
22.Kf2 Kh1 23.Ne5 d3



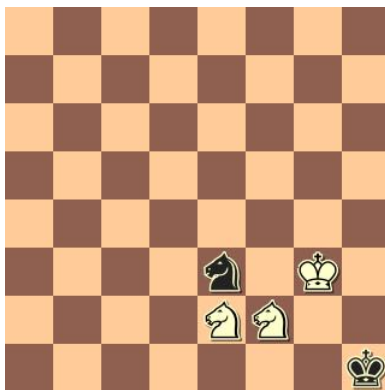
24.Ng4 d2 25.Ne2 d1=N+



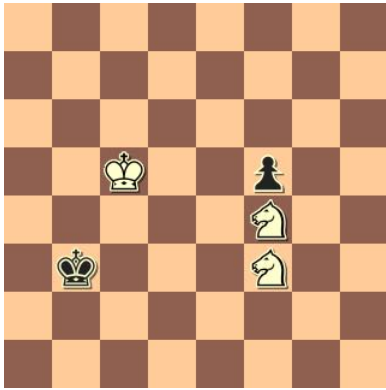
Making a Queen instead by 26...d1=Q allows 27.Ng3#, checkmate.



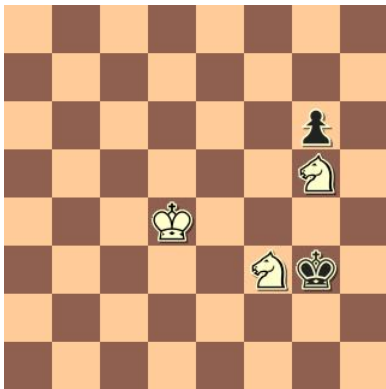
27.Kg3 Ne3 (Or any move.) 28.Nf2#, mate.



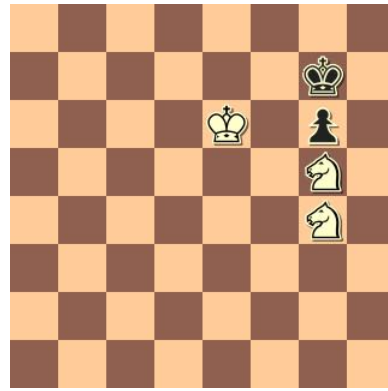
With an f-pawn on the Troitzky Line mate can be enforced only on h8, and by the same process we just saw.



Black is driven up the a-file, then across the 8th rank, and into the fatal corner. Again, if Black lets himself be surrounded in a left side corner then mate happens there, but best defense means a transit along two edges of the board. Troitzky observed that this starting position is very much like the previous d-pawn starting position, but rotated 90 degrees.

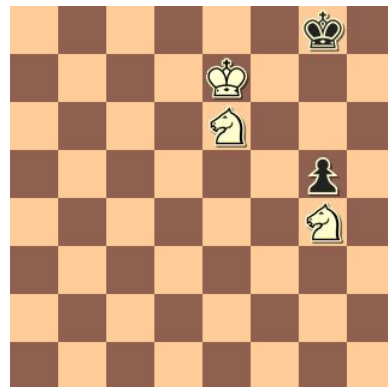


Here we have a g-pawn on the Troitzky Line, and marching the Black King onto h8 is the only way to checkmate by force against correct defense. From the diagram Black will be pushed up the h-file or the f-file, and into the upper right quadrant. Black can be defeated in the other corners also, if he's inaccurate.

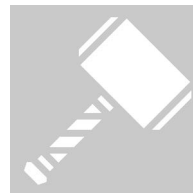
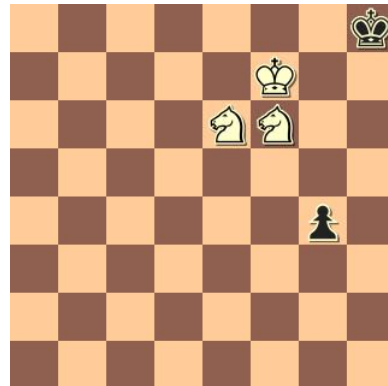


In the end White obtains a position as above, and mate is delivered shortly regardless of who moves first, e.g.

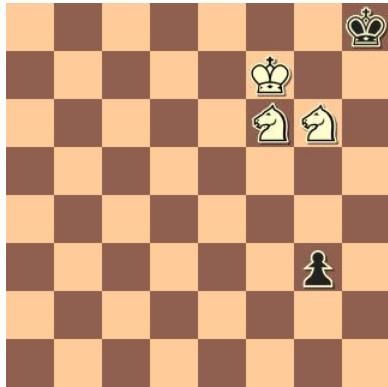
1.Ke7 Kg8 2.Ne6 g5



3.Nf6+ Kh8 4.Kf7 g4

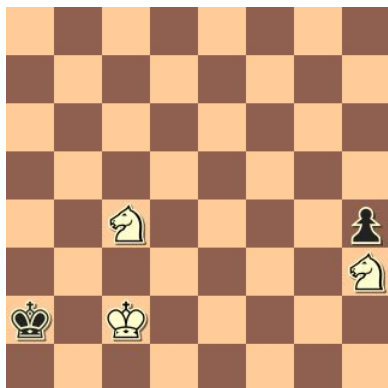


5.Nf4 g3 6.Ng6#, checkmate.



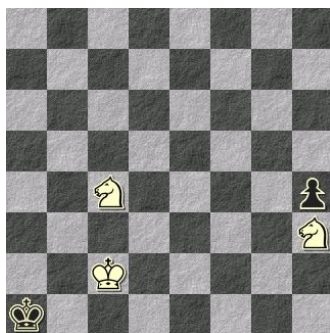
If an h-pawn is on the Troitzky Line, on the h4 square, there is another special case. Once again White can only mate by force at h8 when facing best play - makes sense, because White's reserve Knight on h3 needs but two or three moves to arrive on the scene. If Black falters he can, as usual, be defeated in the other corners, yet even stubborn resistance will lead eventually to the h8 square.

On one condition. What if Black takes refuge in the lower left quadrant?



True, he can't avoid getting trapped soon on the a1-a2 squares, as seen above. And if it is White's turn he can deliver mate in five moves, beginning with 1.Nf4. But what happens if Black has the move?

1...Ka1

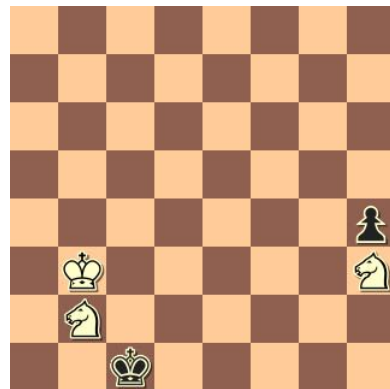


Suddenly that extra Knight on h3 can't get there in time, and no checkmate is in sight.

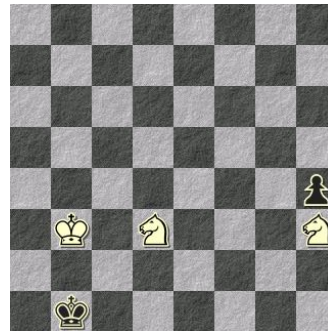
What's worse, White has no way to triangulate or pass the move to Black. The solution to this vexing puzzle is one discovered by Troitzky nearly 125 years ago. The maneuvers are elegant, simple in conception but complex in execution.

White has to re-arrange the corner position, then put his reserve Knight on the f3 square.

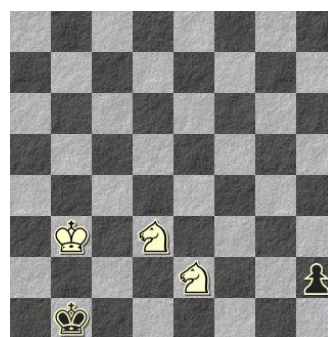
1.Kb3 Kb1 2.Nb2 Kc1



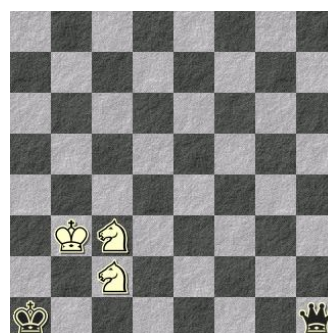
2...Ka1 3.Nd3 Kb1



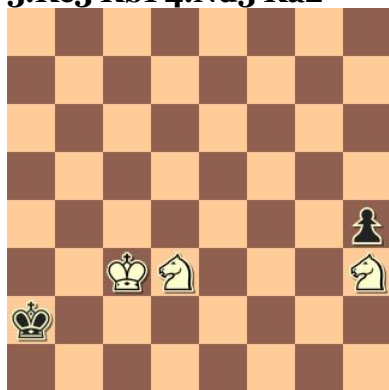
4.Nhf4 h3 5.Ne2 h2



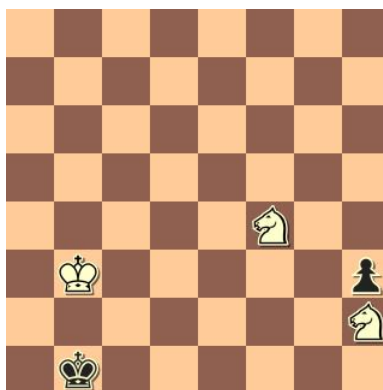
6.Nc3+ Ka1 7.Nb4 h1=Q 8.Nc2#



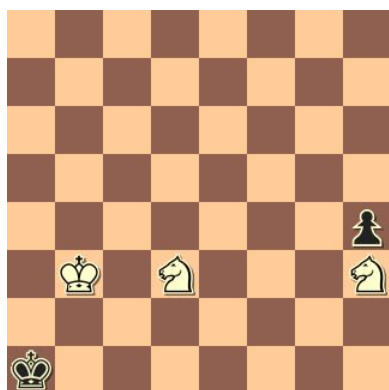
3.Kc3 Kb1 4.Nd3 Ka2



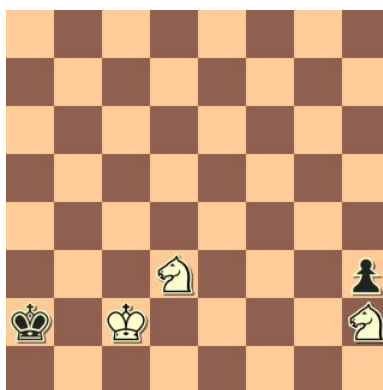
9.Nf4 h3 10.Nh2 Kb1



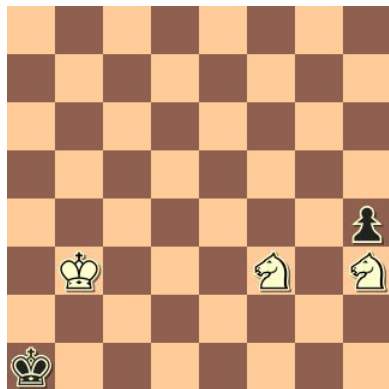
5.Kb4 Kb1 6.Kb3 Ka1



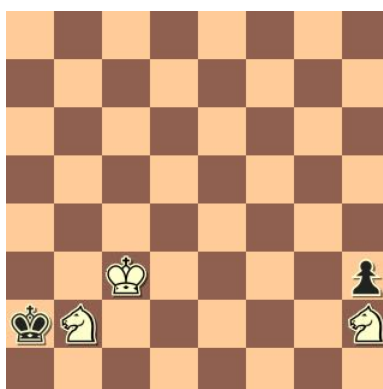
11.Nd3 Ka1 12.Kc2 Ka2



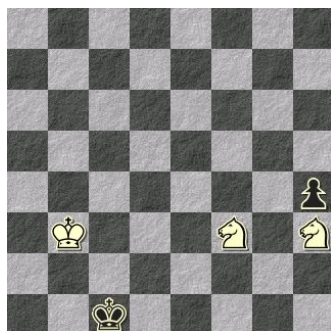
7.Ne5 Kb1 8.Nf3 Ka1



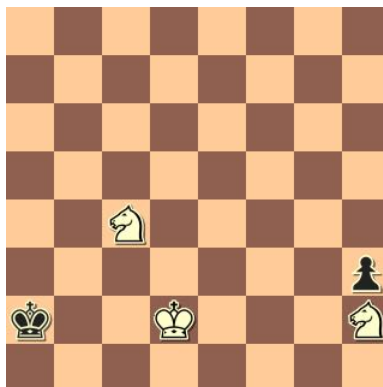
13.Nb2 Ka3 14.Kc3 Ka2



9...Kc1 will transpose to the main line.

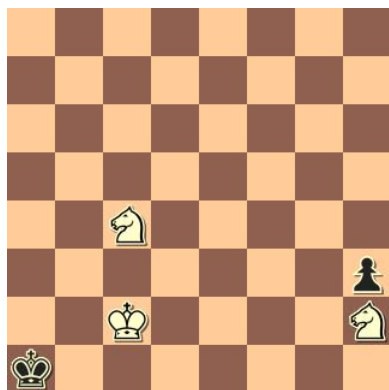


15.Nc4 Kb1 16.Kd2 Ka2



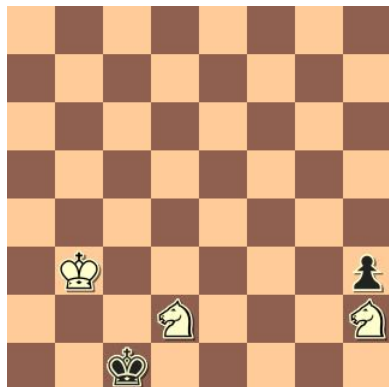
White can now play Nh3-f4, answering the h-pawn advance with Nf3-h2 to freeze it on the h3 square - like so:

17.Kc2 Ka1



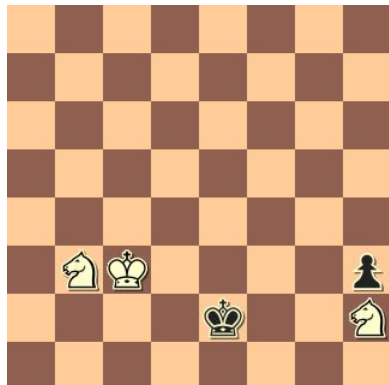
White has reached the original position after Black's first move, except the Kingside pieces have been brought down a rank. Only now, after 18 moves, is White ready to evict the opposite King from his a1 hideaway. Black has caused a lot of trouble, and his fate is to be dragged along the lower rank toward the h1 square.

18.Kb3 Kb1 19.Nd2+ Kc1

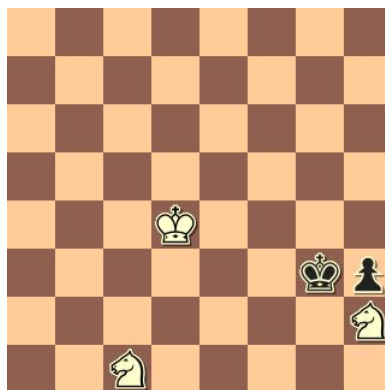


20...Ka1 allows mate in three.

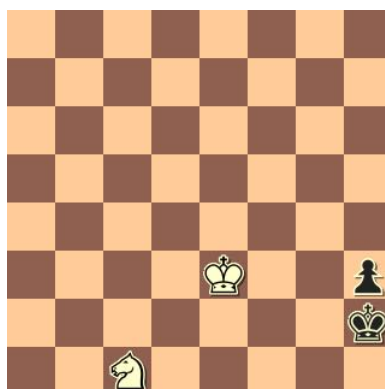
20.Kc3 Kd1 21.Nb3 Ke2



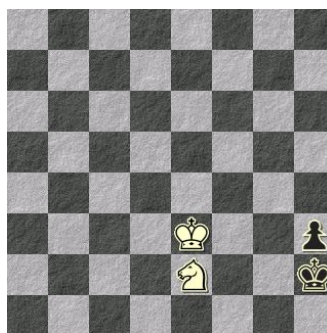
22.Kd4 Kf2 23.Nc1 Kg3



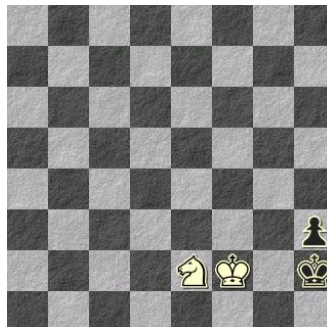
24.Ke3 Kxh2 Black has been forced to capture White's blockading h2 Knight.



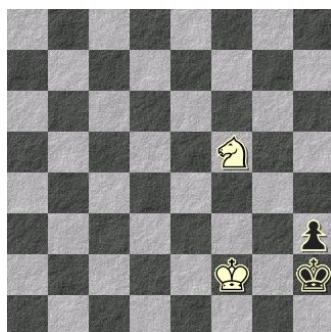
25...Kg2 26.Ne2 Kxh2



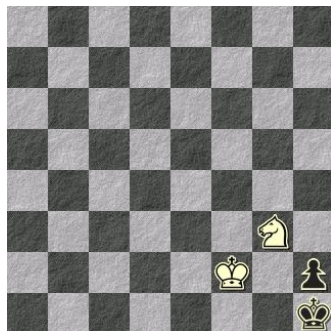
27.Kf3 Kh1 28.Kf2 Kh2



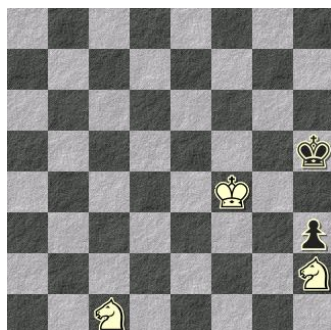
29.Nd4 Kh1 30.Nf5 Kh2



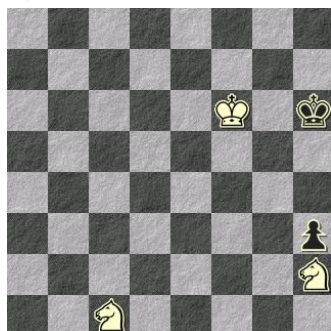
31.Ne3 Kh1 32.Nf1 h2 33.Ng3#



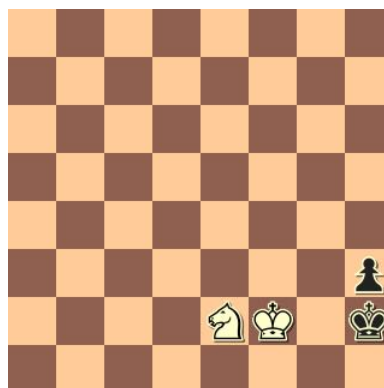
25...Kh4 26.Kf4 Kh5



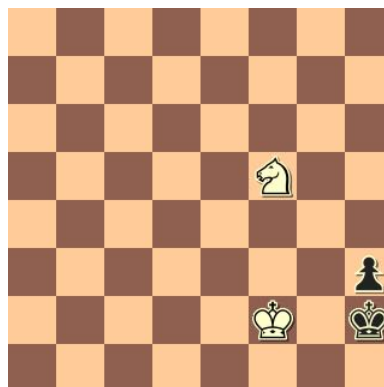
27.Kf5 Kh6 28.Kf6



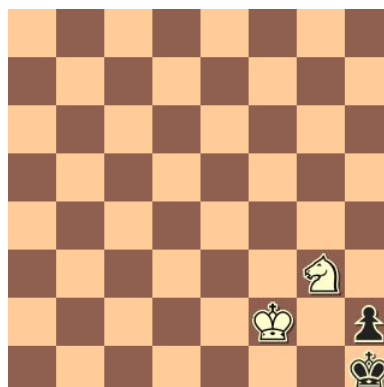
25.Kf2 Kh1 26.Ne2 Kh2



27.Nd4 Kh1 28.Nf5 Kh2

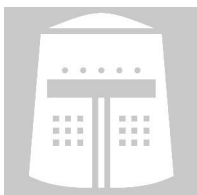


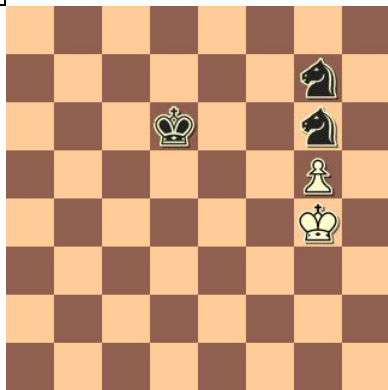
29.Ne3 Kh1 30.Nf1 h2 31.Ng3#



Right then, we've had a glimpse of the Troitzky Line in action! There's certainly more to it than most internet posters want to discuss. So is that all? Have we finished with this strange line on the chess board?

Not so fast. Suppose now that Black's pawn has advanced further than the Troitzky Line allows - is the game therefore drawn? Well, that depends!



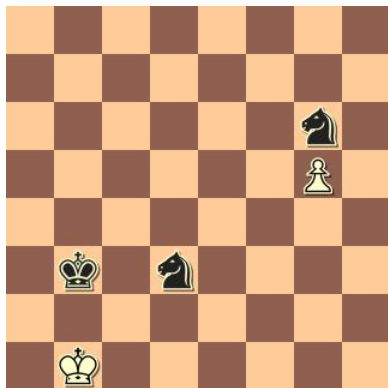


Kohler-Pickard



Above, White has gone two squares beyond the Troitzky Line, as shown from the game Kohler-Pickard, Hot Springs 1985. It's White to play, one move after the final capture, Black having just moved 83...Ng7 to stack the Knights. We've noticed this formation already, and the impenetrable wall of squares covered by these Knights. The square occupied by White at g4 is called the "pit" by Troitzky, and the King can approach no closer to the Knights.

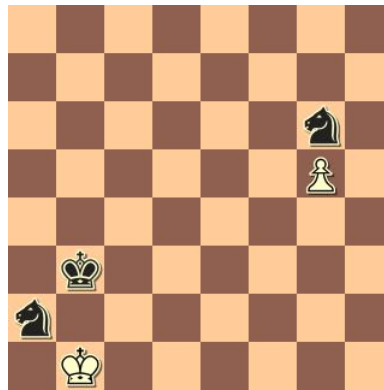
The complete game is [here](#), but after 25 more moves and most of my time gone, the following standard corner position arose.



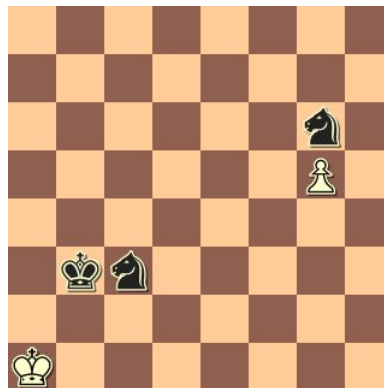
Where's the checkmate? Turns out, Troitzky says the White pawn at g5 means that Black can only force mate in the a8 corner or the h1 corner! The problem with this position is that White will get a new Queen with check. Greatly annoyed, and with about 90 seconds remaining on our analog clock, I went for the draw.



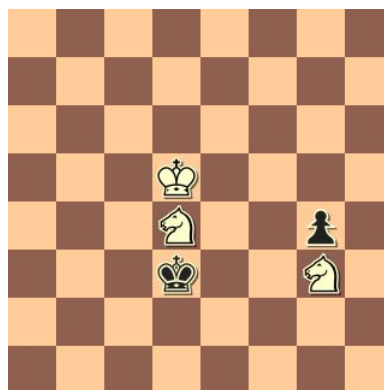
109.Ka1 Nc1 110.Kb1 Na2



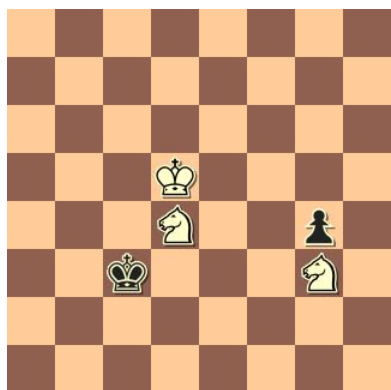
111.Ka1 Nc3, stalemate.



The previous diagram, without 111...Nc3, may still be a win for Black if he can run White's King into a correct corner. I haven't had the heart to check those tablebases! As consolation, however, we can observe Topalov doing things the right way in the game Topalov-Karpov, Monaco 2000, pictured below. We're just after the last capture, and it's a reversed image of the previous example.

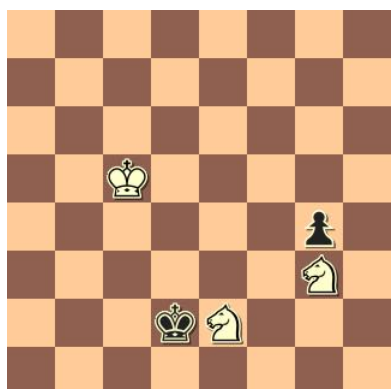


Black to move played **61...Kc3**,

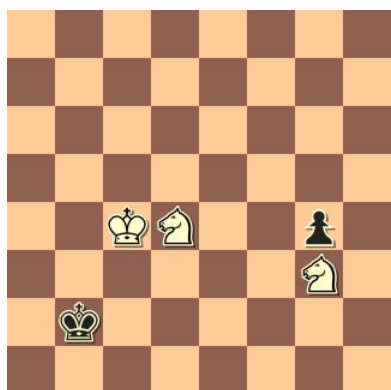


and the game continued

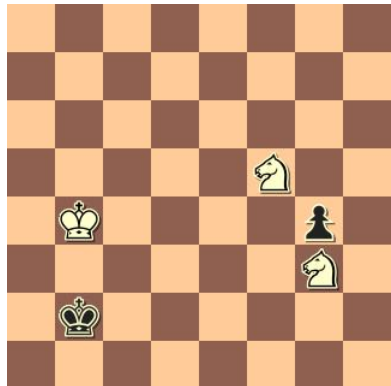
62.Kc5 Kd3 63.Nde2 Kd2



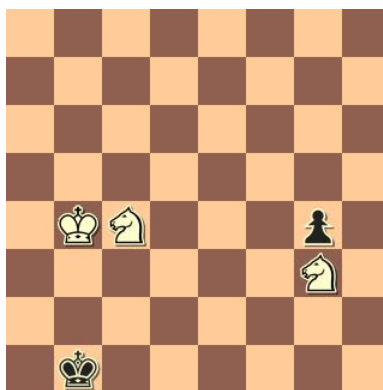
64.Kc4 Kc2 65.Nd4+ Kb2



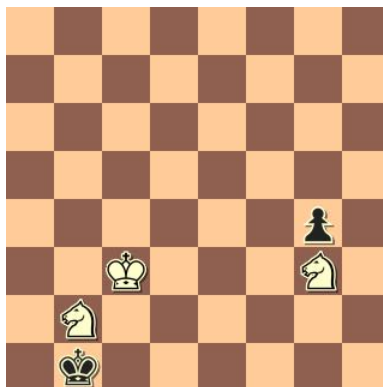
66.Kb4 Ka2 67.Ndf5 Kb2



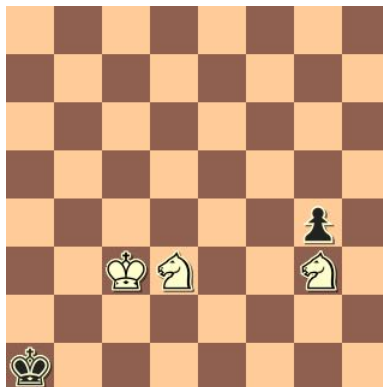
68.Nc3 Ka2 69.Nc4 Kb1



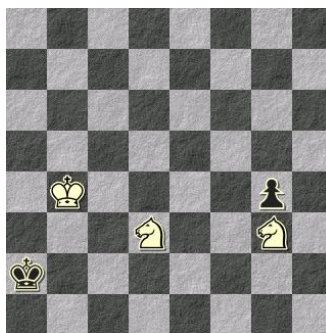
70.Kc3 Kc1 71.Nb2 Kb1

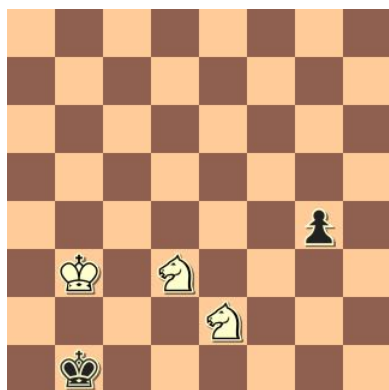


72.Nd3 Ka1

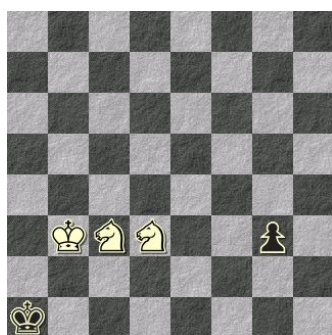


If 72...Ka2 then 73.Kb4, etc.

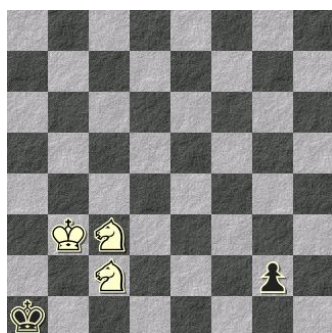


73.Kh3 Kb1 74.Ne2, 1-0

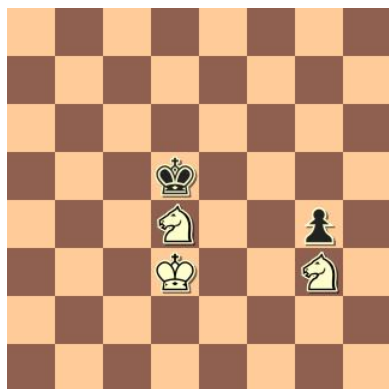
After 74.Ne2 Karpov resigned rather than face 74...g3 75.Nc3+ Ka1



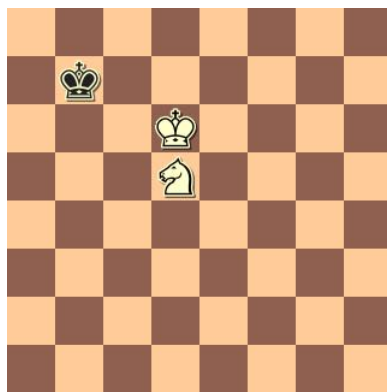
76.Nb4 g2 77.Nc2#, mate.



So yes, there are conditions all along the Troitzky Line - many of them, and likely many more that haven't been found yet. Going back to Topalov's game after the final capture, for instance, and switching around the Kings we get the next diagram.



Here the King positions are reversed, and Black can easily flee to the safe a8 corner. Is White still winning and, if so, how long will it take until checkmate? In fact, can all checkmates from the Troitzky Line be accomplished within the 50-move rule? That's a practical question of no concern to our composer of studies. And answering such questions may risk a leap into mathematics!



If we confine these three pieces above to the upper left quadrant, and exclude ridiculous positions (trapping the Knight, leaving the zone, etc.), still the Knight will have seven or eight "reasonable" moves. Then shift the White King one square, say to d7, and again seven or eight squares appear for the Knight. After all that we can move the Black King one square and start over counting - before rewinding everything and giving Black the first move! So figure that one quadrant alone will yield perhaps 1,500 positions to examine. Of course, that's from somebody who barely passed "Fun with Numbers" in college.

A win or a draw? I don't think a computer will help us.

In conclusion, we understand that chess players are busy with opening preparation and tournament play. It's easy to concentrate on the most likely events, and ignore outliers like two Knights versus a pawn. But do so at your own peril; play enough chess and your time will come!

Troitzky, A., Collection of Studies, Tschaturanga: Ed. Olms, ISBN 3-283-00114-6 (1992). Reprinted by Ishi Press, ISBN 0-923891-10-2 (2006). Included is the supplement on the endgame of two knights against pawns.

Averbakh, Y., Knight Endings, Batsford ISBN 0-7134-0552-X (1977).